Estimating and reporting astrometric uncertainties

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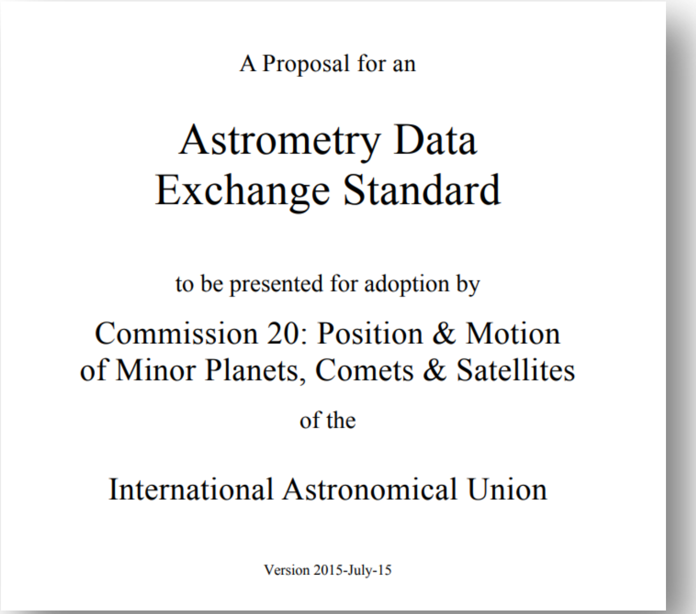
The ADES format

The Minor Planet Center now accepts astrometry submission using the new Astrometry Data Exchange Standard (ADES).

ADES offers a few extremely important capabilities for astrometry submission:

1. The ability to report measured coordinates with many more significant digits, if needed and justified.
2. The ability to report astrometric uncertainties, using the rmsRA and rmsDec keywords.
3. The capability to report a correlation between uncertainties in RA and Dec.


A Proposal for an

Astrometry Data Exchange Standard

to be presented for adoption by

Commission 20: Position & Motion of Minor Planets, Comets & Satellites

of the

International Astronomical Union

Version 2015-July-15

Chesley et al. (2015)

Many observers are now taking advantage of these capabilities, and in particular of uncertainties.

However, the accuracy of the reported quantities is sometimes poor.

→ THE EUROPEAN SPACE AGENCY

An example of underestimated uncertainties

Unfortunately, many observers are reporting uncertainty values that, after an orbital fitting is performed, prove to be significantly underestimated.

As an example, we can look at this table published in the campaign paper for the IAWN timing campaign on 2019 XS (Farnocchia et al., The Planetary Science Journal 3:156 (2022))

Table 3
Fraction of Tracklets with Weighted rms of the Residuals Greater Than 1

Uncertainty	<0.″05	0.″05–0.″1	0.″1–0.″2	0.″2–0.″5	>0.″5
Fraction	100%	73%	62%	39%	3%

Note. A weighted rms > 1 indicates that the estimated uncertainties underestimate the actual extent of the astrometric position errors.

Farnocchia et al. (2022)

The table shows that a large majority of observers reporting uncertainties smaller than 0.2″ underestimated the actual astrometric error of their measurements.

An overview of astrometric uncertainties

In the following, we will briefly discuss the "ingredients" that constitute an astrometric uncertainty, and present a very simple conservative way to assess them.

The uncertainty of an astrometric measurement is the combination of three main components:

1. Centroiding error, how well the "center" of the object detection is determined.
2. Astrometric solution error, the error in the functions that translates pixels into RA and Dec.
3. Any local bias of the astrometric solution due to other sources.

In turn, point 2. is the combination of two basic components:

2a. Errors in the stellar positions from the reference catalog.

2b. Discrepancies between the astrometric solution and the image itself (e.g. field distortions).

We will now discuss how these elements matter in today's astrometry.

The Gaia catalog

In April 2018, ESA released the second version of its Gaia astrometric catalog (DR2), containing extremely accurate astrometric positions and proper motions of more than a billion stars.

Gaia is far superior to any previously existing stellar catalog for:

1. The intrinsic accuracy of the stellar positions.
2. The accuracy of the stellar proper motions.
3. The number of stars present in each typical field.

Gaia / ESA

Using Gaia (DR2, EDR3, DR3 or any future releases) allows us to neglect 2a. and the catalog component of 3. of the list before, making astrometry and error determination much easier.

Table 4
Number of Campaign Observations for Each Star Catalog

Star Catalog	Number of Observations	Fraction	References
Unknown	62	6.5%	
USNO-A2.0	8	0.8%	Monet (1998)
USNO-B1.0	30	3.1%	Monet et al. (2003)
2MASS	36	3.8%	Skrutskie et al. (2006)
UCAC-3	19	2.0%	Zacharias et al. (2010)
UCAC-4	27	2.8%	Zacharias et al. (2013)
Gaia DR1	8	0.8%	Gaia Collaboration et al. (2016a)
Gaia DR2	731	76.4%	Gaia Collaboration et al. (2018)
Gaia EDR3	33	3.4%	Gaia Collaboration et al. (2021)
ATLAS-2	3	0.3%	Tonry et al. (2018)

Farnocchia et al. (2022)

Since the release of Gaia DR2, there is basically no valid reason to use any other astrometric catalog for (optical) asteroid astrometry!

The centroiding

Measuring the position of an object means, first of all, locating it and determining its "center".

For an asteroid, i.e. an object that we can assume to be inactive and unresolved, this is typically done by fitting a PSF (1D Gaussian, 2D Gaussian, trail…) to the object's detection.

Here is a simple "recipe" to estimate how well we can determine that center of the detection (1.), and provide an error bar, under the simple case of a Gaussian PSF.

Under the assumption that the image is properly sampled, ~2 pixels per FWHM of the source, we can estimate the uncertainty as:

$$E_1 = \frac{\theta}{2S}$$

where θ is the source's Full Width Half Maximum (FWHM), and S is its Signal to Noise Ratio (SNR).

When the source is trailed, more sophisticated trail-fitting techniques are needed to properly assess the correlated RA and Dec uncertainties and covariance.

The astrometric solution

The second main contribution to the astrometric uncertainty comes from the astrometric solution.

Gaia allows us to basically neglect the uncertainty and biases of the stellar positions themselves.

However, a few other sources of error remain. Among them:

- How well the astrometric solution matches the actual distortions of the focal plane in the image.
- Chromatic biases (e.g. differential chromatic aberration), displacing each star with respect to the others of a different amount, and therefore creating "noise" in the solution itself.

All these things can be corrected for, but they rarely are. That's not a problem if we can, again, conservatively estimate how much they affect the astrometry, and include that in our error bars.

There is a simple and conservative way to estimate how much these unaccounted effects (2b.) are contributing to our uncertainty: simply using the root mean square (RMS) of the astrometric solution as the second component of our uncertainty, E_2. It can be different for each coordinate of course.

A simple example

We obtain a detection of an asteroid with:

- SNR of the source over the background, $S = 8$
- FWHM of the detection, $\theta = 4"$

Therefore $E_1 = \frac{\theta}{2S} = \frac{4"}{2\times 8} = 0.25"$

Now, the astrometric solution, solved to an order that does not show any clear systematic residuals on the field stars, has:

- RMS in both RA and Dec of $E_2 = 0.13"$

The astrometric error of each coordinate will then be the quadrature sum of the two components:

$$E = \sqrt{E_1^2 + E_2^2} = \sqrt{0.25"^2 + 0.13"^2} = 0.28"$$

We can then, conservatively, round this up to $0.3"$, and submit this uncertainty in ADES.

A side note: geographical coordinates

So far, we've discussed astrometry itself, i.e. the extraction of positional measurements of an asteroid from optical images, and their uncertainties.

However, in some cases the dominant source of uncertainty in an astrometric position does not come from the measurement of the asteroid, but from that of the observer!

Many MPC codes are associated to poorly determined coordinates:

- The coordinates themselves may be old, determined decades ago before the GPS era, and may be wrong by hundreds of meters.
- The observatory itself may be poorly defined, e.g. using a code that corresponds to a distributed site, or to another telescope.
- The reported altitudes may be incorrectly referred to a system that is not the proper WGS84 ellipsoid (never the geoid!).

These issues can all be easily addressed and solved today, and the MPC accepts coordinate corrections for known codes. Please check the coordinates of your site and fix if needed!

Summary

Here are a few recommendation that every observer should follow to submit proper astrometry and error bars to the MPC.

- Use the new ADES format, and in particular report astrometric uncertainties.
- Never use any astrometric catalog that is not Gaia DR2 or subsequent releases.
- Don't trust the astrometric uncertainties produced by your software, unless you are sure you are properly modeling every major aspect. Use the simple conservative rules presented here instead.
- Revise the geographical coordinates of your observatory.

The goal of these suggestions is NOT to be perfect, but to be conservative and not wrong!

THANKS!